Radian/Degree Conversion Formulas:

x deg = $x\cdot\frac{\pi}{180}$ radians $\qquad$ x rad = $x\cdot\frac{180}{\pi}$ degrees

Examples:

$62^\circ = 62\cdot\frac{\pi}{180} = \frac{31}{90}\cdot\pi \approx 1.082$ radians

$\frac{3\pi}{2}$ radians $= \frac{3\pi}{2}\cdot\frac{180}{\pi} = 270$ degrees

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Coterminal angles:

Any angles who have the same terminal side when drawn in standard position are coterminal angles. To find coterminal angles of a given angle, first find the equivalent angle between 0 and 360 degrees (or 0 and 2π radians) and that angle's equivalent angle between −360 and 0. This will give you two angles that are coterminal. All other coterminal angles are a sum of the positive integer multiples of 360 and the positive equivalent angle, or they are a sum of the negative equivalent angle with the negative integer multiples of 360. It's easy if you draw a picture! (If an angle is given in radian measure, just substitute 2π for 360)

Examples:

Find the coterminal angle or angles of each angle:

What are the coterminal angles of $t = 40^\circ$: $-(360 - 40) = -320^\circ$

Other coterminal angles are: $40 + 360n = 400, 760, 1120, ...$

and: $-320 + -360n = -680, -1040, -1400, ...$

What are the coterminal angles of $t = -150^\circ$: $360 - 150) = 210^\circ$

Other coterminal angles are: $210 + 360n = 570, 930, 1290, ...$

and: $-150 + -360n = -510, -870, -1230, ...$

What are the coterminal angles of $t = -800°$

First find the positive and negative equivalent angels between −360 and 360:

Positive equivalent angle: $1080 - 800 = 280°$

Negative equivalent angle: $-(360 - 280) = -80°$

Now find the other coterminal angles:

Other coterminal angles are: $280 + 360n = 640, 1000, 1360, \ldots$

and: $-80 - 360n = -440, -800, -1160, \ldots$

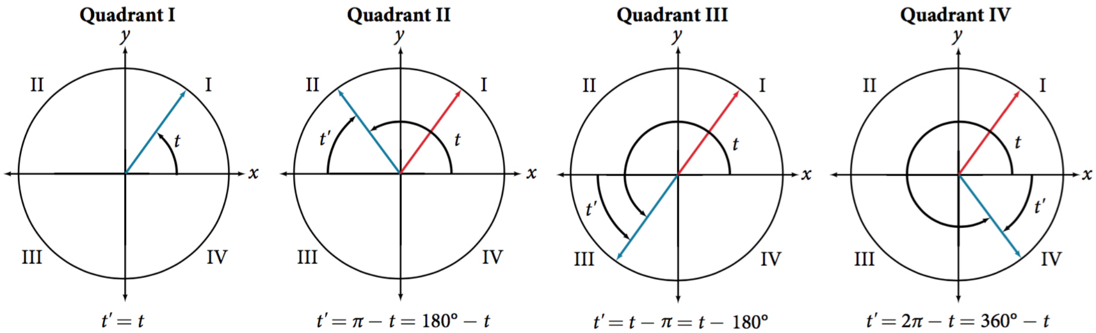
Reference Angles: Use the picture from the text if you get confused. To find the reference angle for any angle less than zero or greater than 360, first convert it to its smallest postive equivalent angle, then use the rules for an angle in each quadrant:

If t is in Q1 ($0 \le t \le 90°$) then $t' = t$

If t is in Q2 ($90 < t \le 180°$) then $t' = \pi - t$ or $t' = 180° - t$

If t is in Q3 ($180 < t \le 270°$) then $t' = t - \pi$ or $t' = t - 180$

If t is in Q4 ($270 < t \le 360°$) then $t' = 2\pi - t$ or $360 - t$



Examples:

What is the reference angle of 75°: QI -> $t' = 360 - 75 = 285°$

What is the reference angle of 160°: QII -> $t' = 180 - 160 = 20°$

What is the reference angle of 230°: QIII -> $t' = 230 - 180 = 50°$

What is the reference angle of 350°: QIV -> $t' = 360 - 250 = 10°$

What is the reference angle of −50°: (replace t with 310°), then QIV -> $t' = 360 - 310 = 50°$

What is the reference angle of −500°: (replace t with 220°), then QIII -> $t' = 220 - 180 = 40°$

Arc Length Formulas:

Let s be the length of an arc, let θ be the measure of the angle of the arc in radians. (If θ is in degrees, convert it to radians before using this formula). Let r be the radius of the circle. Then the following formulas apply:

$s = r \cdot \theta$, (arc length = radius · angle), $\theta = \dfrac{s}{r}$, $r = \dfrac{s}{\theta}$

Area of a sector of a circle, with θ in radians :

$$A = \frac{1}{2} \cdot \theta r^2$$

Linear and Angular Velocity:

If an object is moving along an arc, s, and t is how long it takes something to move along path s, then the linear velocity of the object, v is:

$$v = \frac{s}{t}$$

The following formulas are from our text, pg 589:

angular and linear speed

As a point moves along a circle of radius r, its **angular speed**, ω, is the angular rotation θ per unit time, t.

$$\omega = \frac{\theta}{t}$$

The **linear speed**, v, of the point can be found as the distance traveled, arc length s, per unit time, t.

$$v = \frac{s}{t}$$

When the angular speed is measured in radians per unit time, linear speed and angular speed are related by the equation

$$v = r\omega$$

This equation states that the angular speed in radians, ω, representing the amount of rotation occurring in a unit of time, can be multiplied by the radius r to calculate the total arc length traveled in a unit of time, which is the definition of linear speed.

Below are all the arc/Area/Speed formulas in short form:

$$s = r \cdot \theta \, , A = \frac{1}{2} \cdot \theta r^2 \, , v = \frac{s}{t} \, , \omega = \frac{\theta}{t} \, , v = \frac{s}{t} \, , v = r \cdot \omega$$