

The text definitions of the right triangle trig functions (pg 593-594):

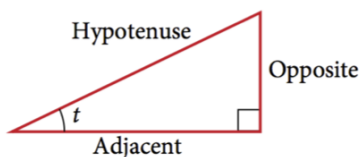


Figure 2 The sides of a right triangle in relation to angle t .

Given a right triangle with an acute angle of t , the first three trigonometric functions are listed.

$$\text{Sine } \sin t = \frac{\text{opposite}}{\text{hypotenuse}} \quad \text{Cosine } \cos t = \frac{\text{adjacent}}{\text{hypotenuse}} \quad \text{Tangent } \tan t = \frac{\text{opposite}}{\text{adjacent}}$$

A common mnemonic for remembering these relationships is SohCahToa, formed from the first letters of “**S**ine is **o**pposite over **h**ypotenuse, **C**osine is **a**djacent over **h**ypotenuse, **T**angent is **o**pposite over **a**djacent.”

For the triangle shown in **Figure 1**, we have the following.

$$\sin t = \frac{y}{1} \quad \cos t = \frac{x}{1} \quad \tan t = \frac{y}{x}$$

Reciprocal Functions

In addition to sine, cosine, and tangent, there are three more functions. These too are defined in terms of the sides of the triangle.

$$\text{Secant } \sec t = \frac{\text{hypotenuse}}{\text{adjacent}} \quad \text{Cosecant } \csc t = \frac{\text{hypotenuse}}{\text{opposite}} \quad \text{Cotangent } \cot t = \frac{\text{adjacent}}{\text{opposite}}$$

Take another look at these definitions. These functions are the reciprocals of the first three functions.

$$\begin{aligned} \sin t &= \frac{1}{\csc t} & \csc t &= \frac{1}{\sin t} \\ \cos t &= \frac{1}{\sec t} & \sec t &= \frac{1}{\cos t} \\ \tan t &= \frac{1}{\cot t} & \cot t &= \frac{1}{\tan t} \end{aligned}$$

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cofunction identities

The **cofunction identities** in radians are listed in **Table 1**.

$\sin t = \cos\left(\frac{\pi}{2} - t\right)$	$\sec t = \csc\left(\frac{\pi}{2} - t\right)$	$\tan t = \cot\left(\frac{\pi}{2} - t\right)$
$\cos t = \sin\left(\frac{\pi}{2} - t\right)$	$\csc t = \sec\left(\frac{\pi}{2} - t\right)$	$\cot t = \tan\left(\frac{\pi}{2} - t\right)$

Example 3 Evaluating Trigonometric Functions of Special Angles Using Side Lengths

Find the exact value of the trigonometric functions of $\frac{\pi}{3}$, using side lengths.

Solution

$$\sin\left(\frac{\pi}{3}\right) = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{3}s}{2s} = \frac{\sqrt{3}}{2}$$

$$\sec\left(\frac{\pi}{3}\right) = \frac{\text{hyp}}{\text{adj}} = \frac{2s}{s} = 2$$

$$\cos\left(\frac{\pi}{3}\right) = \frac{\text{adj}}{\text{hyp}} = \frac{s}{2s} = \frac{1}{2}$$

$$\csc\left(\frac{\pi}{3}\right) = \frac{\text{hyp}}{\text{opp}} = \frac{2s}{\sqrt{3}s} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\tan\left(\frac{\pi}{3}\right) = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{3}s}{s} = \sqrt{3}$$

$$\cot\left(\frac{\pi}{3}\right) = \frac{\text{adj}}{\text{opp}} = \frac{s}{\sqrt{3}s} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$
