# C8S3 Formulas and Notes

On these restricted domains, we can define the inverse trigonometric functions.

- The **inverse sine function** $y = \sin^{-1} x$ means $x = \sin y$. The inverse sine function is sometimes called the **arcsine function**, and notated arcsin$x$.

$$y = \sin^{-1} x \text{ has domain } [-1, 1] \text{ and range } \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

- The **inverse cosine function** $y = \cos^{-1} x$ means $x = \cos y$. The inverse cosine function is sometimes called the **arccosine** function, and notated arccos $x$.

$$y = \cos^{-1} x \text{ has domain } [-1, 1] \text{ and range } [0, \pi]$$

- The **inverse tangent function** $y = \tan^{-1} x$ means $x = \tan y$. The inverse tangent function is sometimes called the **arctangent** function, and notated arctan $x$.

$$y = \tan^{-1} x \text{ has domain } (-\infty, \infty) \text{ and range } \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

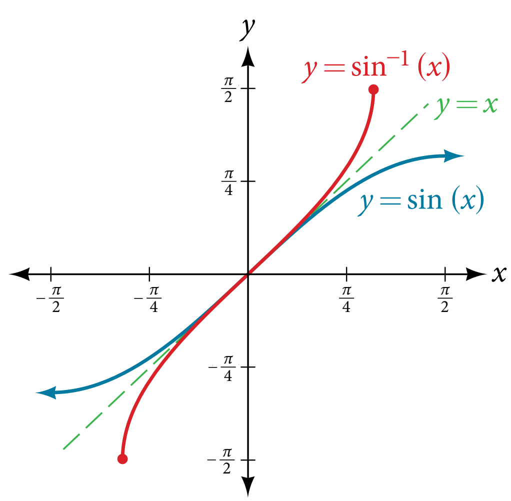


**Figure 4** The sine function and inverse sine (or arcsine) function

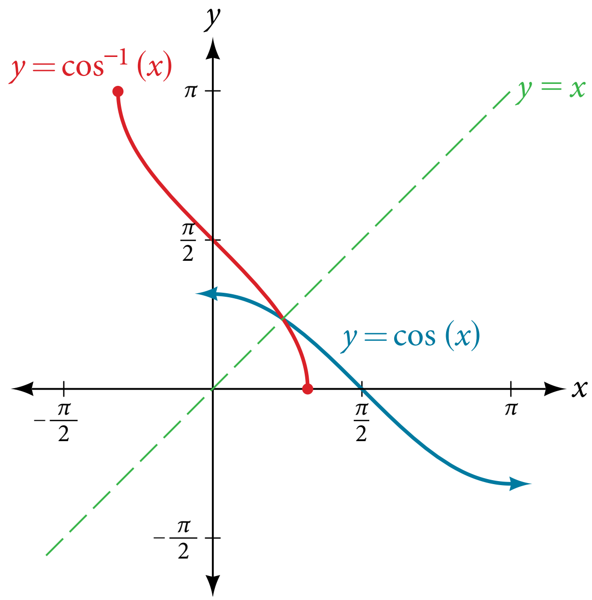


**Figure 5** The cosine function and inverse cosine (or arccosine) function

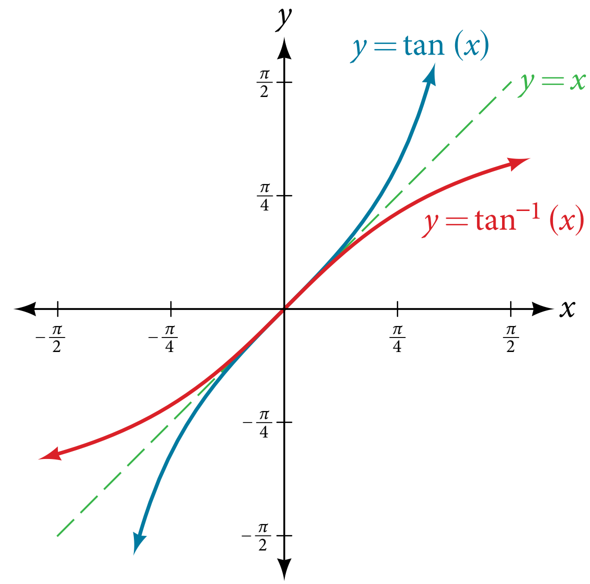


**Figure 6** The tangent function and inverse tangent (or arctangent) function

***relations for inverse sine, cosine, and tangent functions***

For angles in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, if $\sin y = x$, then $\sin^{-1} x = y$.

For angles in the interval $[0, \pi]$, if $\cos y = x$, then $\cos^{-1} x = y$.

For angles in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, if $\tan y = x$, then $\tan^{-1} x = y$.

***compositions of a trigonometric function and its inverse***

$$\sin(\sin^{-1} x) = x \text{ for } -1 \leq x \leq 1$$

$$\cos(\cos^{-1} x) = x \text{ for } -1 \leq x \leq 1$$

$$\tan(\tan^{-1} x) = x \text{ for } -\infty < x < \infty$$

$$\sin^{-1}(\sin x) = x \text{ only for } -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

$$\cos^{-1}(\cos x) = x \text{ only for } 0 \leq x \leq \pi$$

$$\tan^{-1}(\tan x) = x \text{ only for } -\frac{\pi}{2} < x < \frac{\pi}{2}$$

*How To...*

Given functions of the form $\sin^{-1}(\cos x)$ and $\cos^{-1}(\sin x)$, evaluate them.

1. If $x$ is in $[0, \pi]$, then $\sin^{-1}(\cos x) = \frac{\pi}{2} - x$.
2. If $x$ is not in $[0, \pi]$, then find another angle $y$ in $[0, \pi]$ such that $\cos y = \cos x$.

$$\sin^{-1}(\cos x) = \frac{\pi}{2} - y$$

3. If $x$ is in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, then $\cos^{-1}(\sin x) = \frac{\pi}{2} - x$.
4. If $x$ is not in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, then find another angle $y$ in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ such that $\sin y = \sin x$.

$$\cos^{-1}(\sin x) = \frac{\pi}{2} - y$$