What You Really Need to Know to Simplify Rational Expressions with Rational Exponents

1. Property: $x^{r} \cdot y^{s} = x^{(r+s)}$

   Examples: $x^{5} \cdot x^{2} = x^{(5+2)} = x^{7}$, $x^{\frac{3}{4}} \cdot x^{\frac{2}{3}} = x^{(\frac{3}{4}+\frac{2}{3})} = x^{\frac{17}{12}}$

2. Property: $\frac{x^{r}}{y^{s}} = x^{(r-s)}$

   Examples: $\frac{x^{5}}{x^{2}} = x^{(5-2)} = x^{3}$, $\frac{x^{\frac{3}{4}}}{x^{\frac{2}{3}}} = x^{(\frac{3}{4}-\frac{2}{3})} = x^{\frac{1}{12}}$

3. Property: $(x^{r})^{s} = x^{(r \cdot s)}$

   Examples: $(x^{5})^{2} = x^{(5 \cdot 2)} = x^{10}$, $(x^{\frac{2}{3}})^{\frac{3}{4}} = x^{(\frac{3}{4} \cdot \frac{2}{3})} = x^{\frac{1}{2}}$

4. Property: $(x^{r} \cdot y^{s} \cdot z^{t})^{u} = x^{(r)(u)} y^{(s)(u)} \cdot z^{(t)(u)}$

   Example: $(x^{5} \cdot y^{3} \cdot z^{-7})^{4} = x^{(5)(4)} \cdot y^{(3)(4)} \cdot z^{(-7)(4)} \Rightarrow x^{20} \cdot y^{12} \cdot z^{-28}$

5. Property: $x^{-r} = \frac{1}{x^{r}}$

   Example: $x^{-3} = \frac{1}{x^{3}}$

On the remaining pages, you will find examples of simplifying expressions with exponents, starting a simple example and going on from there. If you have trouble simplifying expressions in your class, try finding an example similar to what you are working on. If you can't figure out what's going on after that, you probably need to get some personal help from a tutor, a study buddy, or a teacher / professor.

Examples of Simplifying Some Expressions Involving Exponents:

1. Expression: $2x^2 - 3x^5$

   Simplifiied Expression: $x^2(2 - 3x^3)$

   Explanation: Divide each term in the expression by x^2 to get the factored result.

   For the first term, note that, using property 2 above, $\frac{2x^2}{x^2}$ can be looked at as $2 \cdot \frac{x^2}{x^2} \Rightarrow 2 \cdot x^{2-2} \Rightarrow 2 \cdot x^0 \Rightarrow 2 \cdot 1 = 2$

   For the second term, note that, using property 2 above, $\frac{3x^5}{x^2}$ can be looked at as $3 \cdot \frac{x^5}{x^2} \Rightarrow 3 \cdot x^{5-2} \Rightarrow 3 \cdot x^3 \Rightarrow 3 \cdot 1 = 3x^3$

   A great way to check your work is multiply your answer out and make sure you get what you started with. Note here that $x^2(2 - 3x^3) = 2 \cdot x^2 - 3 \cdot x^3 \cdot x^2 = 2x^2 - 3x^5$.

2. Expression: $x^{\frac{3}{7}} + x^{\frac{4}{5}}$

   Simplifiied Expression: $x^{\frac{3}{7}} \cdot (1 + x^{\frac{13}{35}})$

   Explanation: Divide each term in the expression by $x^{\frac{3}{7}}$ to get the factored result.

   For the first term, note that, using property 2 on the first page, $x^{\frac{3}{7}}$ divided by $x^{\frac{3}{7}}$ is obviously 1.

   For the second term, note that, using property 2 above,

   $x^{\frac{4}{5}}$ divided by $x^{\frac{3}{7}}$ is $x^{\frac{4}{5}-\frac{3}{7}} \Rightarrow x^{(\frac{4}{5}\cdot\frac{7}{7})-(\frac{3}{7}\cdot\frac{5}{5})} \Rightarrow x^{\frac{28}{35}-\frac{15}{35}} \Rightarrow x^{\frac{13}{35}}$

   A great way to check your work is multiply your answer out and make sure you get what you started with. Note here that $x^{\frac{3}{7}} \cdot (1 + x^{\frac{13}{35}}) = x^{\frac{3}{7}} + x^{\frac{4}{5}}$.