

In Exercises 1–10 determine whether or not each of the given equations is exact; solve those that are exact.

1. $(3x + 2y) dx + (2x + y) dy = 0.$
2. $(y^2 + 3) dx + (2xy - 4) dy = 0.$
3. $(2xy + 1) dx + (x^2 + 4y) dy = 0.$
4. $(3x^2y + 2) dx - (x^3 + y) dy = 0.$
5. $(6xy + 2y^2 - 5) dx + (3x^2 + 4xy - 6) dy = 0.$
6. $(\theta^2 + 1)\cos r dr + 2\theta \sin r d\theta = 0.$
7. $(y \sec^2 x + \sec x \tan x) dx + (\tan x + 2y) dy = 0.$
8. $\left(\frac{x}{y^2} + x\right) dx + \left(\frac{x^2}{y^3} + y\right) dy = 0.$
9. $\left(\frac{2s - 1}{t}\right) ds + \left(\frac{s - s^2}{t^2}\right) dt = 0.$
10. $\frac{2y^{3/2} + 1}{x^{1/2}} dx + (3x^{1/2}y^{1/2} - 1) dy = 0.$

Solve the initial-value problems in Exercises 11–16.

11. $(2xy - 3) dx + (x^2 + 4y) dy = 0, \quad y(1) = 2.$
12. $(3x^2y^2 - y^3 + 2x) dx + (2x^3y - 3xy^2 + 1) dy = 0, \quad y(-2) = 1.$
13. $(2y \sin x \cos x + y^2 \sin x) dx + (\sin^2 x - 2y \cos x) dy = 0, \quad y(0) = 3.$
14. $(ye^x + 2e^x + y^2) dx + (e^x + 2xy) dy = 0, \quad y(0) = 6.$
15. $\left(\frac{3 - y}{x^2}\right) dx + \left(\frac{y^2 - 2x}{xy^2}\right) dy = 0, \quad y(-1) = 2.$
16. $\frac{1 + 8xy^{2/3}}{x^{2/3}y^{1/3}} dx + \frac{2x^{4/3}y^{2/3} - x^{1/3}}{y^{4/3}} dy = 0, \quad y(1) = 8.$

17. In each of the following equations determine the constant A such that the equation is exact, and solve the resulting exact equation:

(a) $(x^2 + 3xy) dx + (Ax^2 + 4y) dy = 0.$

(b) $\left(\frac{1}{x^2} + \frac{1}{y^2}\right) dx + \left(\frac{Ax + 1}{y^3}\right) dy = 0.$

Answers to Odd-Numbered Exercises

SECTION 1.1

1. Ordinary; first; linear.
3. Partial; second; linear.
5. Ordinary; fourth; nonlinear.
7. Ordinary; second; linear.
9. Ordinary; sixth; nonlinear.

SECTION 1.2

5. (a) 2, 3, -2. (b) -1, -2, 4.

SECTION 1.3

1. No; one of the supplementary conditions is not satisfied.
3. (a) $y = 3e^{4x} + 2e^{-3x}$. (b) $y = -2e^{-3x}$.
5. $y = 2x - 3x^2 + x^3$.
7. Yes.

SECTION 2.1

1. $3x^2 + 4xy + y^2 = c$.
3. $x + x^2y + 2y^2 = c$.
5. $3x^2y + 2y^2x - 5x - 6y = c$.
7. $y \tan x + \sec x + y^2 = c$.
9. $s^2 - s = ct$.
11. $x^2y - 3x + 2y^2 = 7$.
13. $y^2 \cos x - y \sin^2 x = 9$.
15. $-3y + 2x + y^2 = 2xy$.
17. (a) $A = \frac{3}{2}$; $2x^3 + 9x^2y + 12y^2 = c$.
(b) $A = -2$; $2x^2 - 2y^2 - x = cxy^2$.
19. (a) $x^2y + c$. (b) $2x^{-1}y^{-3} - \frac{3}{2}x^2y^{-4} + c$.
21. (b) x^2 . (c) $x^4 + x^3y^2 = c$.