(13) $(2y\sin x\cos x + y^2\sin x) + (\sin^2 x - 2y\cos x)\frac{dy}{dx} = 0$

I.V.C: $y(0) = 3$

$M = \varphi_x(x,y)$ $\quad N = \varphi_y(x,y)$

Exact? $M_y \stackrel{?}{=} N_x$

$2\sin x\cos x + 2y\sin x \stackrel{?}{=} 2\sin x\cos x + 2y\sin x$ ✓

Find $\varphi(x,y) = \int M\,dx = \int (2y\sin x\cos x + y^2\sin x)\,dx$

$= 2y\int \sin x\cos x\,dx + \int y^2\sin x\,dx$

$u = \sin x$
$du = \cos x\,dx$

$\varphi(x,y) = 2y\cdot\frac{1}{2}\sin^2 x - y^2\cos x + h(y)$

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(13) (cont.)

$$\psi(x,y) = y\sin^2 x - y^2\cos x + h(y)$$

$$\psi_y(x,y) = \sin^2 x - 2y\cos x + h'(y)$$

Set $\psi_y = N$:

$$\sin^2 x - 2y\cos x + h'(y) = \sin^2 x - 2y\cos x$$

$\Rightarrow h'(y) = 0.$

Find $h(y)$: $h(y) = \int 0\,dy = k_1.$

So $\psi = y\sin^2 x - y^2\cos x + k_1$

Now, $\psi = k_2$ since $\dfrac{d(\psi(x,y))}{dx} = 0$

$\therefore\ y\sin^2 x - y^2\cos x + k_1 = k_2$

$\Rightarrow$ Implicit Solution is: $y\sin^2 x - y^2\cos x = K$

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Find K from I.V.C:

$y(0)=3 \Rightarrow$ when $x=0,\ y=3 \Rightarrow$

$3\sin^2(0) - 3^2\cos(0) = K$

$\Rightarrow \quad -9 = K.$

So the specific implicit solution is:

$$y\sin^2 x - y^2\cos x = -9$$

or $y^2\cos x - y\sin^2 x = 9$