(15) $\left(\frac{3-y}{x^2}\right) + \left(\frac{y^2-2x}{xy^2}\right)\frac{dy}{dx} = 0$, $y(-1) = 2$

$M = \varphi_x$ $N = \varphi_y$

$M = \frac{1}{x^2}(3-y)$

$N = \frac{1}{x} - \frac{2}{y^2}$

Exact?: $M_y \stackrel{?}{=} N_x$

$-\frac{1}{x^2} = -\frac{1}{x^2}$ ✓ Yes

Find $\varphi(x,y) = \int M\,dx = \int (3-y)\cdot x^{-2}\,dx$

$= (3-y)(-x^{-1}) + h(y)$

$\varphi(x,y) = \frac{(y-3)}{x} + h(y)$ or $\frac{y}{x} - \frac{3}{x} + h(y)$

Find $\varphi_y = \frac{1}{x} + h'(y)$.

Set $\varphi_y = N \Rightarrow \frac{1}{x} + h'(y) = \frac{1}{x} - \frac{2}{y^2}$

$\Rightarrow h'(y) = -\frac{2}{y^2}$

(next page)

(15) (cont)...

Find $h(y) = \int h'(y)\,dy = \int \left(-\frac{2}{y^2}\right)dy = \int -2y^{-2}\,dy$

$= 2y^{-1} + k_1$

$$h(y) = \frac{2}{y} + k_1 .$$

$$\Rightarrow \Psi(x,y) = \frac{y}{x} - \frac{3}{x} + \frac{2}{y} + k_1$$

$$= \frac{y}{x} - \frac{3}{x} + \frac{2}{y} + k_1 .$$

But $\Psi(x,y) = \frac{y}{x} - \frac{3}{x} + \frac{2}{y} + k_1 = k_2$ $\left(\text{since } \frac{d(\Psi(x,y))}{dx} = 0\right)$

$\Rightarrow$ General Implicit Solution is:

$$\frac{y}{x} - \frac{3}{x} + \frac{2}{y} = K .$$

(next page)

Use I.V.C. to find K:

$y(-1) = 2 \Rightarrow$ when $x = -1$, $y = 2$

$$\therefore \quad \frac{(2)}{(-1)} - \frac{3}{(-1)} + \frac{2}{2} = K$$

$$\Rightarrow \quad -2 + 3 + 1 = K$$

$$\Rightarrow \quad K = 2.$$

$\therefore$ Specific Implicit Solution is:

$$\frac{y}{x} - \frac{3}{x} + \frac{2}{y} = 2$$

or $$y - 3 + \frac{2x}{y} = 2x$$

or $$y^2 - 3y + 2x = 2xy$$