Short Explanation of Completing the Square to Find the Equation of a Circle

First: We know that the graph of an equation in this form:

$$(x - h)^2 + (y - k)^2 = r^2$$

is a circe with center at the point (h,k) and radius r.

Example:

For this equation, $(x - 3)^2 + (y + 5)^2 = 49$,

the graph is a circle with center ( 3 , -5 ) and radius 7.

Second: What if the equation looks like this:

$$x^2 - 8x + y^2 - 6y = 11$$

How do we re-write the equation to easily see if it is a circle and what the radius and center is?

That is what the technique of "Completing the Square" does. Do it like this:

1. Rearrange the terms like this:

$x^2 - 8x \quad + y^2 - 6y \quad = 11$ *(so that you can add a number after the x term and after the y term).*

2. Take half the x term coefficient, square it and add it after the x term.

half of 8 is 4. 4 squared is 16. Add this.

Take half the y term coefficient, square it and add it after the y term.

half of 6 is 3. 3 squared is 9. Add this.

Add both of those numbers to the other side of the equation to keep things equal.

Here is what you get:

$$x^2 - 8x + 16 + y^2 - 6y + 9 = 11 + 16 + 9$$

Now, combine the numbers on the right and factor the terms on the left, like this:

$$(x - 4)^2 + (y - 3)^2 = 36$$

So this is a circle with center ( 4 , 3 ) and radius 6.