Example of using elementary row operations to find the inverse of a 3×3 matrix.

Problem: If $A = \begin{bmatrix} 1 & 4 & 2 \\ -3 & 2 & 1 \\ -2 & -1 & 3 \end{bmatrix}$ find A^{-1}

(note: this is problem 8 on page 276)

Work:

Notes

Step 1:

$$\left[\begin{array}{ccc|ccc} 1 & 4 & 2 & 1 & 0 & 0 \\ -3 & 2 & 1 & 0 & 1 & 0 \\ -2 & -1 & 3 & 0 & 0 & 1 \end{array}\right]$$

Augment A with the 3x3 identity matrix

Step 2:

$$\left[\begin{array}{ccc|ccc} 1 & 4 & 2 & 1 & 0 & 0 \\ [3] & [12] & [6] & [3] & [0] & [0] \\ -3 & 2 & 1 & 0 & 1 & 0 \\ -2 & -1 & 3 & 0 & 0 & 1 \end{array}\right]$$

(Multiply row 1 by 3 and write the result here

You are going to add these two rows together to get the new middle row in step 3.

Step 3:

$$\left[\begin{array}{ccc|ccc} 1 & 4 & 2 & 1 & 0 & 0 \\ 0 & 14 & 7 & 3 & 1 & 0 \\ [2] & [8] & [4] & [2] & [0] & [0] \\ -2 & -1 & 3 & 0 & 0 & 1 \end{array}\right]$$

Multiply Row 1 by 2 And write the result here.

Add these two Rows together to get the new bottom Row in step 4.

Step 4:

$$\left[\begin{array}{ccc|ccc} 1 & 4 & 2 & 1 & 0 & 0 \\ 0 & 14 & 7 & 3 & 1 & 0 \\ 0 & 7 & 7 & 2 & 0 & 1 \\ [0] & [1] & [1] & [2/7] & [0] & [1/7] \end{array}\right]$$

Swap Rows 2 And 3 to get the 1 where you want it for step 5.

Divide this Row by 7

Step 5:

$$\left[\begin{array}{ccc|ccc} 1 & 4 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 2/7 & 0 & 1/7 \\ 0 & 14 & 7 & 3 & 1 & 0 \end{array}\right]$$

You now have your "1" in the second diagonal position, so we will use this Row to get the zeroes Above + below that "1" in step 6.

Step 6:

$$\left[\begin{array}{ccc|ccc} 1 & 4 & 2 & 1 & 0 & 0 \\ [0] & [-4] & [-4] & [-8/7] & 0 & [-4/7] \\ 0 & 1 & 1 & 2/7 & 0 & 1/7 \\ [0] & [-14] & [-14] & [-4] & [0] & [-2] \\ 0 & 14 & 7 & 3 & 1 & 0 \end{array}\right]$$

(multiply by -4

(multiply by -14)

Step 7:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & -1/7 & 0 & -4/7 \\ 0 & 1 & 1 & 2/7 & 0 & 1/7 \\ 0 & 0 & -7 & -1 & 1 & -2 \\ [0] & [0] & [1] & [1/7] & [-1/7] & [2/7] \end{array}\right]$$

divide this row by -7 to get A "1" in the last diagonal position. Use this As the new bottom row in step 8

Step 8:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & -1/7 & 0 & -4/7 \\ 0 & 1 & 1 & 2/7 & 0 & 1/7 \\ 0 & 0 & 1 & 1/7 & -1/7 & 2/7 \end{array}\right]$$

Use this row to zeroes Above it in step 9.

Step 9:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & -\frac{1}{7} & 0 & -\frac{4}{7} \\ 0 & 1 & 1 & \frac{2}{7} & 0 & \frac{1}{7} \\ [0] & [0] & [-1] & [-\frac{1}{7}] & [\frac{1}{7}] & [-\frac{2}{7}] \\ 0 & 0 & 1 & \frac{1}{7} & -\frac{1}{7} & \frac{2}{7} \end{array}\right]$$

Add these two Rows to get youre new middle Row for step

multiply by -1

tep 10:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & -\frac{1}{7} & 0 & -\frac{4}{7} \\ [0] & [0] & [2] & [\frac{2}{7}] & [-\frac{2}{7}] & [\frac{4}{7}] \\ 0 & 1 & 0 & \frac{1}{7} & \frac{1}{7} & -\frac{1}{7} \\ 0 & 0 & 1 & \frac{1}{7} & -\frac{1}{7} & \frac{2}{7} \end{array}\right]$$

Add these two Rows to get your new top Row for step 11.

multiply by 2

tep 11:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{7} & -\frac{2}{7} & 0 \\ 0 & 1 & 0 & \frac{1}{7} & \frac{1}{7} & -\frac{1}{7} \\ 0 & 0 & 1 & \frac{1}{7} & -\frac{1}{7} & \frac{2}{7} \end{array}\right]$$

this is A^{-1}! (see step 12 on the next page)

Step 10:

$$A^{-1} = \begin{bmatrix} 1/7 & -2/7 & 0 \\ 1/7 & 1/7 & -1/7 \\ 1/7 & -1/7 & 2/7 \end{bmatrix}$$

We usually will factor out the $1/7$ to write A^{-1} like this:

$$A^{-1} = \frac{1}{7} \begin{bmatrix} 1 & -2 & 0 \\ 1 & 1 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

← best way to write your answer

You could also write A^{-1} using decimals, rounded off to several places, but this is not as accurate.

$$A^{-1} = \begin{bmatrix} 0.143 & -0.286 & 0 \\ 0.143 & 0.143 & -0.143 \\ 0.143 & -0.143 & 0.286 \end{bmatrix}$$